12th Financial Risks International Forum

Sabes D., Monfort A., Pegoraro F. and Renne J.-P. (2018)
Affine Term Structure Models with Stochastic Lower Bound

Eberlein E., Gerhart C. and Grbac Z. (2018)
Multiple Curve Levy Forward Price Model Allowing for Negative Interest Rates

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Comments on

Affine Term Structure Models with Stochastic Lower Bound an Application to Euro-Area OIS Rates

Sabes, Monfort, Pegoraro, and Renne (2018)


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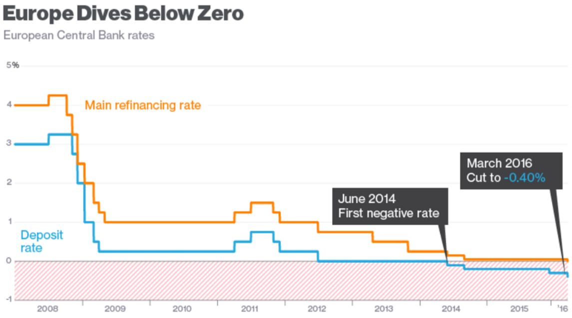
Affine Term Structure Models

Fact

This paper introduces an original term structure model featuring a negative stochastic lower bound (SLB) at which the short rate may be stuck for some time.

This paper has three main contributions

1. The model is compatible with the recent advent of **negative interest rates**,
2. Extends the **zero-lower bound** (ZLB) theoretical literature,
3. Belonging to the affine class, the model is **tractable**.

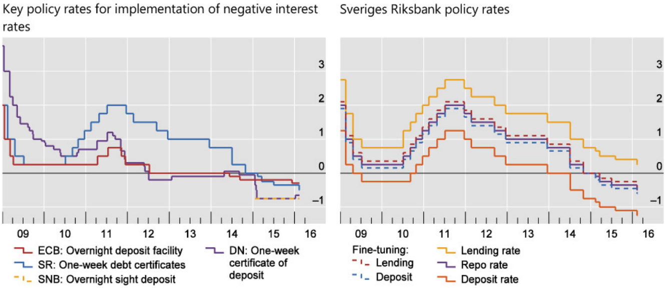


Source: European Central Bank

Affine Term Structure Models

Central bank policy rates

In per cent — Graph A



DN = Danmarks Nationalbank; ECB = European Central Bank; SNB = Swiss National Bank; SR = Sveriges Riksbank.

Source: Bech and Malkhozov (BIS Quaterly Review, 2016)

Affine Term Structure Models

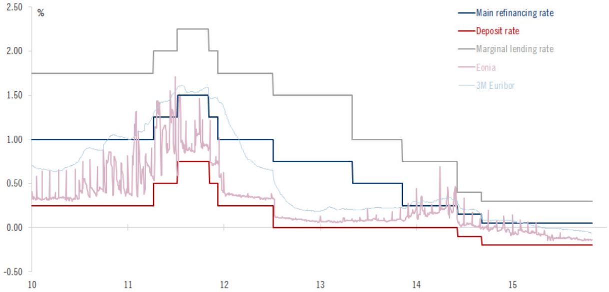
Zero-lower bound (ZLB) litterature

1. Many of the **tractable** yield-curve models are not consistent with the existence of a lower bound for nominal interest rates.
2. Term-structure models designed to accommodate the existence of a lower bound for nominal yields are generally **not tractable.**
3. They are generally based on a shadow-rate s_t approach (Black, 1995) with

$$r_t = \max\left(s_t, \underline{r}\right)$$

with $\underline{r}$ a **constant** lower bound (i.e. Christensen and Rudebusch, JFE 2015).

Affine Term Structure Models



Source: Q&A on the ECB's negative rates, Perspectives Pictet, October 2015

Theoretical contributions

1. This paper specifies a **stochastic process** for the lower bound, which is negative and features as a step-function.
2. Extension of Monfort, Pegoraro, Renne, and Roussellet (2017), based on Vector Autoregressive Gamma (VARG) processes,
3. The factors (including SLB) are able to stay at zero during some periods without zero being an absorbing state.
4. These processes are **affine**, and thus offer closed form formulas for yields, forecasts and conditional variances.

General comments

- The contribution is very convincing and the paper is very well written.
- High probability to be published in top finance journal and to be highly cited.

SLB process

The SLB is equal to the negative Deposit Facility Rate (defined as multiple of -10 basis points), with Y_t an integer-valued random variable:

$$SLB_t = -10Y_t \quad Y_t \in \mathbb{N}$$

The change in Y_t is the sum of two opposite forces

$$\Delta Y_t = \Delta N_t^+ - \Delta N_t^-$$

$$\Delta N_t^+ \big| \Omega_{t-1} \sim \mathcal{P}\left(\lambda_t\right)$$

$$\Delta N_t^- \big| \Omega_{t-1} \sim \mathcal{P}\left(\eta Y_{t-1}\right)$$

Question 1: positiveness of the SLB process

As the change in Y_t is defined as the difference between two Poisson processes, what are the conditions to ensure the non-negativity of Y_t?

- If Y_{t-1} is equal to 0, then ΔN_t^- is equal zero and therefore Y_t cannot be negative
- Otherwise, the sign of ΔY_t, and thus of $SLB = -10Y_t$, depends on Y_{t-1}, η and λ_t.
- Without *additional conditions*, I think that there is a probability (even it is small in practice) to get a **positive** SLB.

Question 2: intensity processes

$$SLB_t = -10Y_t \text{ with } \Delta Y_t = \Delta N_t^+ - \Delta N_t^-$$

$$\Delta N_t^+ \big| \Omega_{t-1} \sim \mathcal{P}(\lambda_t) \quad \lambda_t \in \mathbb{R}^+$$

$$\Delta N_t^- \big| \Omega_{t-1} \sim \mathcal{P}(\eta Y_{t-1}) \quad \eta > 0, \quad Y_{t-1} \in \mathbb{N}$$

- A **continuous** non-negative intensity process λ_t for the positive forces ΔN_t^+ on the SLB changes.
- A **discrete** intensity process ηY_{t-1} for the negative forces ΔN_t^- on the SLB changes.
- What is the intuition?

Question 3: stationarity

- The estimation method requires a stationary SLB process (?)
- I think that the changes in the SLB process are stationary. Does the stationarity of order 2 is sufficient, here?

Comments on

Multiple Curve Levy Forward Price Model Allowing for Negative Interest Rates

Eberlein, Gerhart, and Grbac (2018)

Summary

- The authors develop a framework for discretely compounding interest rates which is based on the forward price process.
- The quantities that are modeled here, rather than the short rate (like in the previous paper or in the HJM framework), are a set of forward rates.
- The main advantage of the model is that it does not require an arbitrary choice or a statistical estimation of the lower boundary for negative values.

General comments

- Technical paper, relatively hard to read for **non-specialists**.
- However, the authors motivate the usefulness of their approach.
- There is no conclusion in the current version of the draft, and the paper finishes with the calibration.

Multiple Curve Levy Forward Price Model

Question 1: Why the forward process approach allows in a natural way for negative interest rates? What is the intuition?

- Page 9, Equation 2.5, you mention that "*the drift term is specified in such a way that the forward process is a local martingale, to ensure that the model is free of arbitrage*".
- How this no-arbitrage condition is compatible with negative interest rates?

Question 2: For the calibration, you consider two model variants *a* and *b*. What is the difference between the two models?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